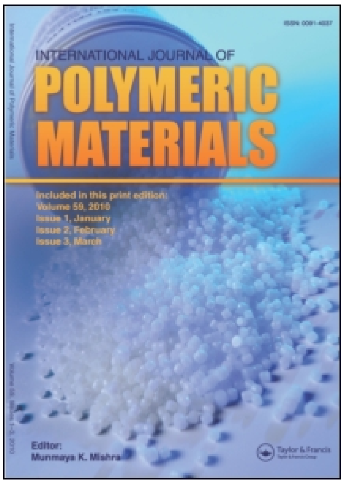

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Analysis of Theories in Computer Programs for Estimation of Conveyor Belt Curing Time. Part I. Equivalent Thickness Theory

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The Lepetov and Sahand methods for estimation of conveyor belt curing times are described and analysed. The equivalent thickness theory and extent of errors associated with it in this respect are discussed.

KEY WORDS Curing times, theory, computer programs.

NOTATION

K = Temperature coefficient of vulcanisation
L = Total belt thickness (mm)
L_{eq} = Total equivalent thickness of reference material (mm)
Li = Thickness of replaced material (mm)
L_r = Thickness of reference material (mm)
X = Distance from the top of the conveyor belt (mm)
T_0 = Ambient temperature (°C)
T_1 = Top plate temperature (°C)
T_2 = Bottom plate temperature (°C)
t_e = Equivalent curing time (min)
T_e = Equivalent curing temperature (°C)
t_r = Experimental curing time of reference material (min)
T_r = Experimental curing temperature of reference material (°C)
$T(t)$ = Temperature at centre of homogenized belt as a function of time
$T(X, t)$ = Temperature as a function of distance and time
T_w = Top or bottom plate temperature (°C)
α = Thermal diffusivity (m^2/hr)
αi = Thermal diffusivity of replaced material (m^2/hr)
α_r = Thermal diffusivity of reference material (m^2/hr)

INTRODUCTION

A research project entitled "Analysis of Heat Transfer and Curing Time in Conveyor Belts" was recently initiated at the research and development department of Sahand Rubber Industries Co. for the purpose of estimating optimum curing times for conveyor belts manufactured at this company.

Scarcity of published works in this field was clearly noticeable and the only reference of significance was that of Lepetov *et al.*[1] In this article, for the purpose of determining curing time, the authors considered the following assumptions:

(i) heat transfer is of conductive type and one dimensional (in thickness direction only),
(ii) equal top and bottom plate temperatures,
(iii) homogenization of the belt[2] using relations:

$$L_{eq} = \sum_{i=1}^{N} L_i[\mathrm{sqr}(\alpha_{eq}/\alpha_i)] \tag{1}$$

and

$$L = L_r + L_{eq} \tag{2}$$

(iv) lowest temperature and consequently lowest amount of curing occurs at the centre of the part,
(v) temperature at centre of the part is calculated using equation:

$$T(t) = T_w - (4/\pi)(T_w - T_0)\exp[(-\pi^2/L^2)\alpha t] \tag{3}$$

(vi) use of the following equation for determining the amount of equivalent curing at the centre of the part[3]:

$$t_e = t_r K^{[(T_r - T_e)/10]} \tag{4}$$

(vii) temperature coefficient of vulcanisation is constant and equal to 2,
(viii) the cure is complete when sum of the calculated curing times for the centre of the belt equals the experimentally determined time for 90% cure, and
(xi) thermal diffusivity coefficients of carcass and covers are constant and independent of temperature.

The model studied by Lepetov *et al.* had the following characteristics:

Top cover thickness, 3 mm
Carcass thickness, 11.1 mm
Bottom cover thickness, 1 mm
Experimental time for 90% cure of constituents, 20 min
Thermal diffusivity of constituents, 3.44e−4 m^2/hr

It must be emphasized that the Lepetov method is not applicable for the case of different top and bottom plate temperatures. For this reason we have attempted modification of the method and the result is a new approach that we call the Sahand method, and which is utilized at our company for estimation of conveyor belt curing time.

SAHAND METHOD

Nonsteady-state one-dimensional heat transfer equation may be expressed as:

$$\delta T/\delta t = \alpha\delta^2 T/\delta X^2 \tag{5}$$

which, neglecting second and higher order terms, has solution:

$$T(X, t) = T_1 - (T_1 - T_2)X/L - (4/\pi)[(T_1 + T_2)/2 - T_0] \cdot \sin(\pi X/L)\exp[(-\pi^2/L^2)\alpha t] \tag{6}$$

In our computer program for estimation of curing time, the carcass thickness in relations (1) and (2) is replaced by an equivalent top cover thickness and temperature distribution in top and bottom covers, at any instant, determined by means of Equation (6). Furthermore, T_1 and T_2 are assumed to be the momentary temperatures of points at a distance from the top of the belt equal to the top cover thickness, and of points at a distance from the bottom of the belt equal to the bottom cover thickness respectively, and used for determining temperature distribution within the carcass. That is, in relation (6), α and L represent the thermal diffusivity and thickness of carcass respectively, and T_0 is the previous temperature of the same surface at any instant. Using the computed temperature and by means of Equation (4) equivalent curing times for carcass and top and bottom covers are then calculated separately. The procedure is repeated after addition of time intervals, and by comparing the sum of equivalent curing times thus obtained with the experimentally determined time for 90% cure of the constituents, the time corresponding to optimum cure of all points within the belt is computed.

The following are assumed in this program:

(i) thermal diffusivity is constant and independent of temperature,
(ii) temperature coefficient of vulcunisation is constant and independent of temperature,
(iii) at any instant, upper and lower surface temperatures of carcass are constant.

Thermal diffusivity and temperature coefficient of vulcanisation can be determined experimentally.

EQUIVALENT THICKNESS THEORY

In the conveyor belt studied by Lepetov *et al.*, theoretical replacement of total thickness of top and bottom covers (4 mm) by an equivalent 3.64 mm of carcass, as calculated from Equation (1), yields an equivalent belt of thickness 14.64 mm made up entirely of carcass material. The point of least cure is thus at belt centre at a distance of 7.32 mm from either surface. This was a special case with carcass much thicker than covers and in which the constituents had equal experimental optimum curing times.

In order to compare and contrast Lepetov with our method in the light of equivalent thickness theory, we examined theoretical models of belts with various thickness and assumed total comparison, and calculated curing times using procedures outlined in Table I.

In all cases top and bottom cover materials had identical properties. Data used in computations are shown in Table II. The results are presented in Table III and shown graphically in Figure 1 as variation of percent error in curing times determined by procedures 2, 3 and 4 relative to procedure 1, with thickness ratio (the ratio of sum of top and bottom cover thicknesses to that of the carcass).

SEPARATE TREATMENT OF CARCASS

In our method in order to determine temperature at various points within the carcass, the carcass is treated separately. That is, T_1 and T_2 are assumed to be the temperature of top cover-carcass and bottom cover-carcass interfaces, respectively. To examine the amount of error produced by this assumption, we determined temperature distribution of a theoretical model in two ways. In the first, the model is assumed to be monolithic, and in the second, it is presumed to consist of top cover carcass and bottom cover, and temperature distribution in the carcass section at any instant determined separately.

It must be emphasized that in the second method the constituents are assumed to be of the same material with identical properties. Program input data is listed in Table IV.

TABLE I

Procedure used for calculation of curing times of theoretical belts

Procedure	Assumed total composition	Determination(s)	Method
1	bottom cover	temperature distribution	Sahand
2	carcass	temperature distribution	Sahand
3	top cover	temperature distribution & degree of cure	Lepetov
4	carcass	temperature distribution & degree of cure	Lepetov

TABLE II

Data used for computation of curing time

Thermal diffusivity of top cover	3.33e-4 m^2/hr
Thermal diffusivity of carcass	3.53e-4 m^2/hr
Temperature coefficient of vulcanisation of top cover	1.76
Temperature coefficient of vulcanisation of carcass	1.72
Initial temperature	40 °C
Top plate temperature	160 °C
Bottom plate temperature	150 °C
Experimental temperature for 90% cure of top cover	158 °C
Experimental time for 90% cure of top cover	7.5 min
Top cover reversion time	25 min
Experimental temperature for 90% cure of carcass	158 °C
Experimental time for 90% cure of carcass	5.9 min
Carcass reversion time	25 min
Point intervals	0.5 min

Variation of computed temperature differences for the carcass with time are shown in Figures 2-1 and 2-2. Results for the top and bottom covers were insignificant and negligible and are not shown here. Figure 3 illustrates percent error in calculated temperature difference of points at the top cover-carcass interface. Due to the symmetry of the model and equal top and bottom plate temperatures, identical results were also obtained for the bottom cover-carcass interface. Optimum curing times obtained by both methods are similar and equal to 14 min.

LIMITS IN THE CHOICE OF TIME INTERVAL

It is seen from Table III that unequal time intervals were used in our program for estimation of curing times. This is due to limits in the choice of time interval as

TABLE III

Comparison of procedures for estimation of curing time

Time interval (min)	Top cover thickness (mm)	Carcass thickness (mm)	Bottom cover thickness (mm)	Thickness ratio [a]	Curing time P_1[b] (min)	Curing time P_2 (min)	Curing time P_3 (min)	Curing time P_4 (min)	Percent error P_2 relative to P_1	Percent error P_3 relative to P_1	Percent error P_4 relative to P_1
0.5	0.5	5.0	0.5	0.20	12.5	12.5	12.5	10.0	0	0	20
1.0	1.0	10.0	1.0	0.20	17.0	17.0	17.0	14.0	0	0	18
1.0	3.0	9.0	1.5	0.50	17.0	17.0	18.0	16.0	0	6	6
0.5	1.5	6.0	1.5	0.50	14.0	14.0	14.0	11.5	0	0	18
0.5	1.5	4.0	1.5	0.75	13.0	13.0	13.0	10.5	0	0	19
0.5	2.0	4.0	2.0	1.00	13.5	13.5	13.5	11.0	0	0	19
0.5	2.5	5.0	2.5	1.00	14.5	14.5	14.5	12.5	0	0	14
1.0	3.0	6.0	3.0	1.00	17.0	17.0	17.0	14.0	0	0	18
1.0	3.5	7.0	3.5	1.00	19.0	19.0	19.0	17.0	0	0	11
1.0	4.0	8.0	4.0	1.00	21.0	21.0	22.0	19.0	0	5	10
0.5	3.0	3.5	1.5	1.30	13.5	13.5	13.5	11.0	0	0	19
1.0	4.0	5.0	3.5	1.50	17.0	17.0	17.0	15.0	0	0	12
1.0	3.5	4.0	3.5	1.75	16.0	16.0	16.0	14.0	0	0	13
1.0	4.0	4.5	5.0	2.00	19.0	19.0	19.0	16.0	0	0	16
0.5	3.0	2.0	3.0	3.00	13.5	13.5	13.5	11.0	0	0	19
1.5	8.0	3.0	7.0	5.00	25.5	25.5	25.5	22.5	0	0	12
1.0	5.0	2.0	5.0	5.00	17.0	17.0	17.0	15.0	0	0	12
1.5	8.0	1.0	7.0	15.00	22.5	22.5	22.5	19.5	0	0	13

a : Ratio of sum of top and bottom cover thicknesses to carcass thickness.

b : P_i stands for ith procedure.

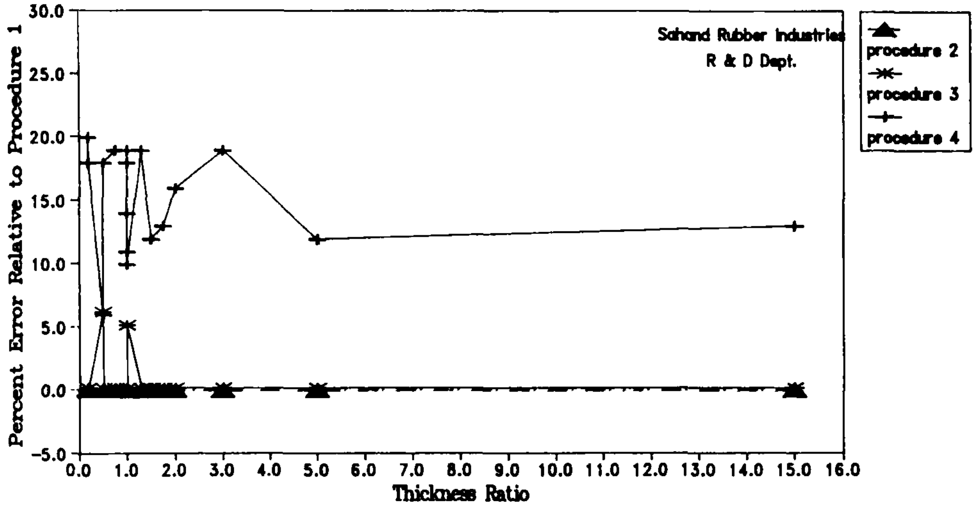

FIGURE 1 Comparison of procedures for homogenization of conveyor belt.

TABLE IV

Program input data

Top cover thickness	3 mm
Carcass thickness	4 mm
Bottom cover thickness	3 mm
Experimental time for 90% cure	7 min
Experimental temperature for 90% cure	160 °C
Thermal diffusivity	3e-4 m^2/hr
Temperature coefficient of vulcanisation	1.9
Top and bottom plate temperatures	160 °C
Initial temperature	40 °C
Time interval	0.5 min

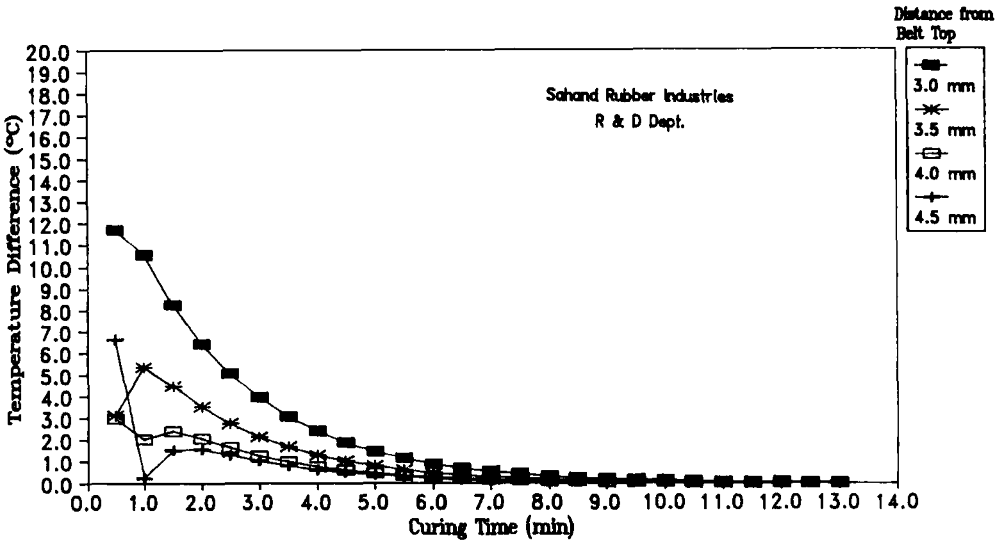

FIGURE 2-1 Variation of temperature difference with curing time for points within carcass determined by Lepetov and Sahand methods (point distance 3.0 to 4.5 mm).

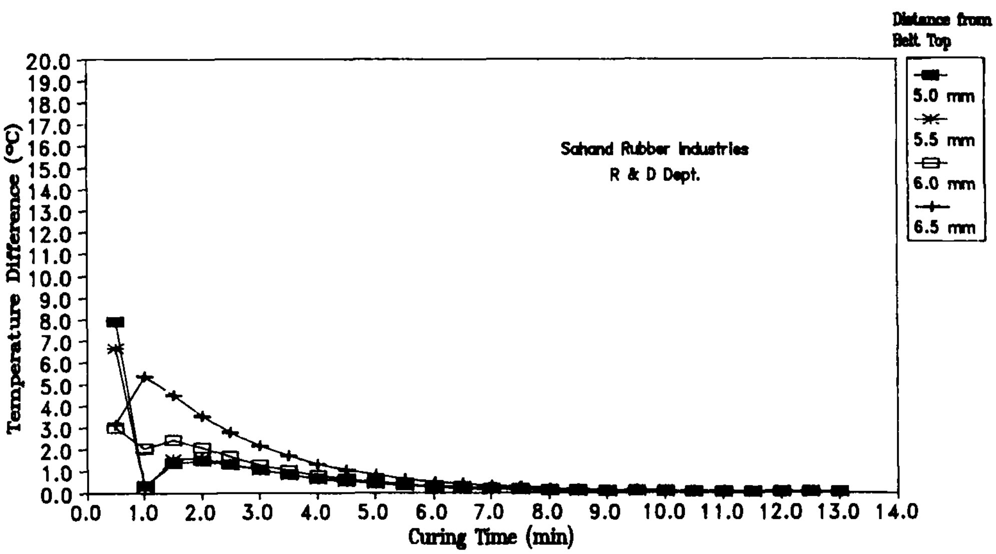

FIGURE 2-2 Variation of temperature difference with curing time for points within carcass determined by Lepetov and Sahand methods (point distance 5.0 to 6.5 mm).

input data for the program. Although small values increase the precision of calculations, they also cause the program to become divergent for the case of relatively thick belts, thus yielding no answers. This may be explained as follows.

Heat transfer equation for the case of equal top and bottom plate temperatures becomes:

$$T(X, t) = T_w - (4/\pi)(T_w - T_0)\sin(\pi X/L)\exp[(-\pi^2/L^2)\alpha t] \tag{7}$$

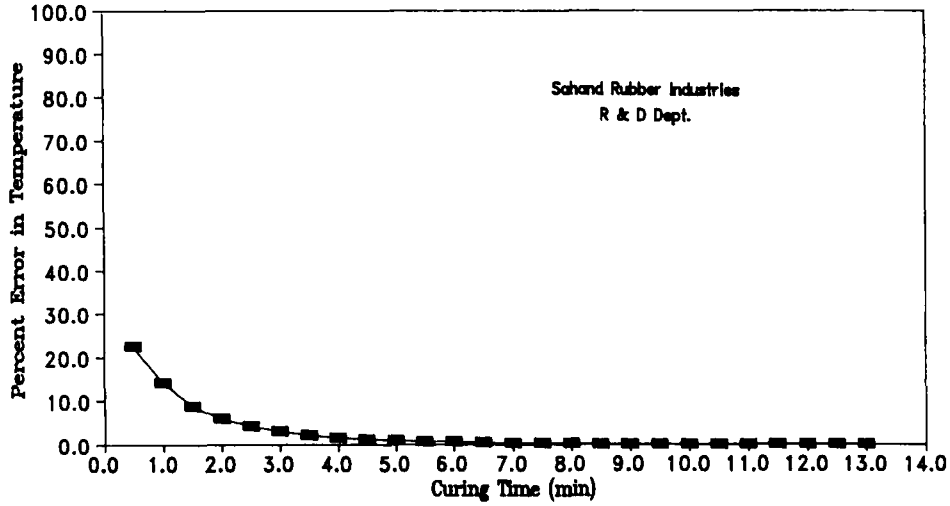

FIGURE 3 Percent error in calculated temperature.

or

$$[T_w - T(X, t)]/(T_w - T_0) = (4/\pi)\sin(\pi X/L)\exp[(-\pi^2/L^2)\alpha t] \quad (8)$$

At the begining of cure since $T(X, t) = T_0$, the left hand side of Equation (8) will be equal to 1 and tending to zero with time. The required condition for the program to be convergent is, therefore, that the left hand side of Equation (8) must equal or be smaller than 1, that is

$$(4/\pi)\sin(\pi X/L)\exp[(-\pi^2/L^2)\alpha t] \leq 1 \quad (9)$$

or

$$\exp[(-\pi^2/L^2)\alpha t] \leq (\pi/4)[1/\sin(\pi X/L)] \quad (10)$$

hence

$$(-\pi^2/L^2)\alpha t \leq \ln(\pi/4) - \ln[\sin(\pi X/L)] \quad (11)$$

therefore

$$t \geq [L^2/(\alpha\pi^2)]\{-\ln(\pi/4) + \ln[\sin(\pi X/L)]\} \quad (12)$$

The term $\ln[\sin(\pi X/L)]$ is always negative with maximum value of zero when $\sin(\pi X/L) = 1$ or $X = L/2$, therefore:

$$t \geq [L^2/(\alpha\pi^2)][-\ln(\pi/4)] \quad (13)$$

or

$$t \geq 0.0245L^2/\alpha \tag{14}$$

so that in the case of equal top and bottom plate temperatures the minimum required time interval for the program to remain convergent can be calculated from Equation (14).

Now considering unequal plate temperatures, we have:

$$T(X, t) = T_1 - (T_1 - T_2)X/L - (4/\pi)[(T_1 + T_2)/2 - T_0]\sin(\pi X/L)\exp[(-\pi^2/L^2)\alpha t] \tag{15}$$

dividing by T_0

$$T(X, t)/T_0 = T_1/T_0 - [(T_1 - T_2)/T_0]X/L - (4/\pi)[(T_1 + T_2)/2T_0 - 1]\sin(\pi X/L)\exp[(-\pi^2/L^2)\alpha t] \tag{16}$$

At the beginning of cure the left hand side of Equation (16) is equal to 1 and tending to zero with time so that the required condition is

$$T_1/T_0 - [(T_1 - T_2)/T_0]X/L - (4/\pi)[(T_1 + T_2)/2T_0 - 1] \cdot \sin(\pi X/L)\exp[(-\pi^2/L^2)\alpha t] \geq 1 \tag{17}$$

or

$$(T_1 - T_0)/T_0 - [(T_1 - T_2)/T_0]X/L \geq (4/\pi)[(T_1 + T_2)/2T_0 - 1] \cdot \sin(\pi X/L)\exp[(-\pi^2/L^2)\alpha t] \tag{18}$$

giving

$$\exp[(-\pi^2/L^2)\alpha t] \leq \{[T_1 - T_0]/T_0 - [(T_1 - T_2)/T_0]X/L\}/\{(4/\pi)[(T_1 + T_2)/2T_0 - 1]\sin(\pi X/L)\} \tag{19}$$

hence

$$(-\pi^2/L^2)\alpha t \leq \ln\{[(T_1 - T_0)/T_0 - [(T_1 - T_2)/T_0]X/L]/[(4/\pi)[(T_1 + T_2)/2T_0 - 1]\sin(\pi X/L)]\} \tag{20}$$

therefore

$$t \geq -[L^2/(\alpha\pi^2)]\ln\{[(T_1 - T_0)/T_0 - [(T_1 - T_2)/T_0]X/L]/[(4/\pi)[(T_1 + T_2)/2T_0 - 1]\sin(\pi X/L)]\} \quad (21)$$

Equation (21) is more complex than Equation (7) and the maximum value of t can no longer be considered to occur at $X = L/2$. For this reason in our program we calculated the right hand side of Equation (21) for values of X ranging from $L/100$ to L at increments of $L/100$, taking the highest value obtained as the minimum time interval required for the program to remain convergent.

DISCUSSION

Table III and Figure 1 provide evidence that the equivalent thickness theory, when used solely for the estimation of temperature distribution, is quite accurate and error free. Using a constant value for α/L^2, the computed temperature difference is the same irrespective of whether the top and bottom covers are assumed to be of the carcass material or vice versa. This is because curing of each component is evaluated separately using data relevant to it and therefore the theory is exact within the limits of use of our method.

In contrast, for the simultaneous determination of temperature distribution and degree of curing of the homogenized belt (procedures 3 and 4), appreciable errors are introduced by this theory if certain points are not observed. It must be noted that in substituting equivalent thicknesses the magnitude of the thickness is not a deciding factor. This is evident from Figure 1 which shows instances in which the carcass is much thicker than other components and replacement of top and bottom covers with carcass gives higher error in curing time compared to the replacement of carcass with top and bottom material. The reason lies in the different experimental curing times of the constituents which is the most important factor determining the form of equivalent thickness replacement. Obviously, the component possessing highest experimental time for 90% cure should be employed for replacement. This is exemplified by substitution of carcass with top cover which has the highest curing time resulting in lower errors (procedure 3 in comparison to procedure 1).

In cases where the experimental curing times are similar or very close, the thickness factor should be taken into consideration and the thickest constituent used for homogenization.

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